SABR BASED VOLATILITY SURFACE PARAMETERIZATION

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1. SABR Based Volatility Surface Parameterization for Swaptions

The calibration of the quoted swaption smiles via SABR (using the model defined by Equations (5.4)-(5.6) in [1]) gives the possibility to produce parsimonious parameterizations, where the whole volatility cube can be represented by four different matrices (Table 1) one per SABR parameter (usually the ATM matrix replaces the SABR based α; this is done to improve clarity and match the way in which the brokers quote ATM implied volatilities). This provides a great amount of simplification as the whole swaption matrix can be tracked by simply looking at three matrices (if we exclude the Beta one, which is generally kept fixed). In the same way, SABR based risk sensitivities (cf. [1], Section 5.6) can be projected on the same four matrices, providing a very clear yet complete allocation of risks across expiry/tenor as well as level/skew/curvature.

ATM (%)

Expiry/Tenor	$1y$	$2y$	$3y$	$4y$	$5y$	$6y$	...	$10y$
$1y$	50	48	48	43	43	37	$\cdots$	35
$2y$	55	53	53	50	50	44	$\cdots$	42
$3y$	60	58	58	55	55	54	$\ddots$	52
$4y$	60	58	58	55	55	54	$\cdots$	52
$5y$	58	56	56	53	53	52	$\cdots$	50
$6y$	58	56	56	53	53	52	$\cdots$	50
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$10y$	53	51	51	48	48	47	$\cdots$	51

Rho (%)

Expiry/Tenor	$1y$	$2y$	$3y$	$4y$	$5y$	$6y$	...	$10y$
$1y$	60	57	57	55	55	53	$\cdots$	50
$2y$	55	52	52	50	50	48	$\cdots$	45
$3y$	50	47	47	45	45	43	$\ddots$	40
$4y$	50	47	47	45	45	43	$\cdots$	40
$5y$	45	42	42	40	40	38	$\cdots$	35
$6y$	45	42	42	40	40	38	$\cdots$	35
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$10y$	40	37	37	35	35	33	$\cdots$	30

Vol of Vol (%)

Expiry/Tenor	$1y$	$2y$	$3y$	$4y$	$5y$	$6y$	...	$10y$
$1y$	44	43	43	41	41	40	$\cdots$	38
$2y$	39	38	38	36	36	35	$\cdots$	33
$3y$	35	34	34	32	32	31	$\ddots$	29
$4y$	35	34	34	32	32	31	$\cdots$	29
$5y$	32	31	31	29	29	28	$\cdots$	26
$6y$	32	31	31	29	29	28	$\cdots$	26
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$10y$	25	24	24	22	22	21	$\cdots$	19

Beta (%)

Expiry/Tenor	$1y$	$2y$	$3y$	$4y$	$5y$	$6y$	...	$10y$
$1y$	30	30	30	30	30	30	$\cdots$	30
$2y$	30	30	30	30	30	30	$\cdots$	30
$3y$	30	30	30	30	30	30	$\ddots$	30
$4y$	30	30	30	30	30	30	$\cdots$	30
$5y$	30	30	30	30	30	30	$\cdots$	30
$6y$	30	30	30	30	30	30	$\cdots$	30
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$10y$	30	30	30	30	30	30	$\cdots$	30

TABLE 1. SABR parameterized volatility cube.

2. SABR Based Volatility Surface Parameterization for Caps and Floors

The same volatility surface parameterization introduced for swaptions in the previous section, can be extended to include also the SABR parameters calibrated to the cap/floor market (using the model defined by Equations (5.1)-(5.3) in [1]), as caplets/floorlet can be though of single period swaptions. Practically this allows us to extend the LIBOR swaption volatility cube showed in Table 1 to include also the cap/floor market, resulting in the volatility parameterization dispayed in Table 2 (where the red highlighted regions correspond to caplets/floorlets). On top of the positive features described in the previous section, SABR allows to characterize and describe, as well as keep track of the whole interest rate volatility complex in an aggregated way.

A final note for the cases in which a shifted SABR approximation is used for the calibration of the cap/floor and swaption markets (cf. [1], Section 5.10). As the SABR calibrated parameters as well as the risk figures are projected on the matrices showed in Table 2, it is clear that using different shift sizes in different sections of the market would lead to SABR calibrated paramenters and risk figures which are not consistent between themselves. If not handled carefully, this could lead to hedging leakeage.

ATM (%)

Expiry/Tenor	$3m$	$6m$	$1y$	$2y$	$3y$	$4y$	$5y$	$6y$	$\dots$	$10y$
$1y$	50	50	50	48	48	43	43	37	$\cdots$	35
$2y$	55	55	55	53	53	50	50	44	$\cdots$	42
$3y$	60	60	60	58	58	55	55	54	$\ddots$	52
$4y$	60	60	60	58	58	55	55	54	$\cdots$	52
$5y$	58	58	58	56	56	53	53	52	$\cdots$	50
$6y$	58	58	58	56	56	53	53	52	$\cdots$	50
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$10y$	53	53	53	51	51	48	48	47	$\cdots$	51

Rho (%)

Expiry/Tenor	$3m$	$6m$	$1y$	$2y$	$3y$	$4y$	$5y$	$6y$	$\dots$	$10y$
$1y$	60	60	60	57	57	55	55	53	$\cdots$	50
$2y$	55	55	55	52	52	50	50	48	$\cdots$	45
$3y$	50	50	50	47	47	45	45	43	$\ddots$	40
$4y$	50	50	50	47	47	45	45	43	$\cdots$	40
$5y$	45	45	45	42	42	40	40	38	$\cdots$	35
$6y$	45	45	45	42	42	40	40	38	$\cdots$	35
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$10y$	40	40	40	37	37	35	35	33	$\cdots$	30

Vol of Vol (%)

Expiry/Tenor	$3m$	$6m$	$1y$	$2y$	$3y$	$4y$	$5y$	$6y$	$\dots$	$10y$
$1y$	44	44	44	43	43	41	41	40	$\cdots$	38
$2y$	39	39	39	38	38	36	36	35	$\cdots$	33
$3y$	35	35	35	34	34	32	32	31	$\ddots$	29
$4y$	35	35	35	34	34	32	32	31	$\cdots$	29
$5y$	32	32	32	31	31	29	29	28	$\cdots$	26
$6y$	32	32	32	31	31	29	29	28	$\cdots$	26
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$10y$	25	25	25	24	24	22	22	21	$\cdots$	19

Beta (%)

Expiry/Tenor	$3m$	$6m$	$1y$	$2y$	$3y$	$4y$	$5y$	$6y$	$\dots$	$10y$
$1y$	30	30	30	30	30	30	30	30	$\cdots$	30
$2y$	30	30	30	30	30	30	30	30	$\cdots$	30
$3y$	30	30	30	30	30	30	30	30	$\ddots$	30
$4y$	30	30	30	30	30	30	30	30	$\cdots$	30
$5y$	30	30	30	30	30	30	30	30	$\cdots$	30
$6y$	30	30	30	30	30	30	30	30	$\cdots$	30
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$10y$	30	30	30	30	30	30	30	30	$\cdots$	30

TABLE 2. SABR parameterized volatility cube. The red highlighted sections of the matrices represent caplets/floorlets.

References

[1] Crispoldi, C., Wigger, G., and Larkin, P., SABR and SABR LIBOR Market Models in Practice, With Examples Implemented in Python, 1st Edition, Palgrave Macmillan, 2015